Completing the Square

For example:
Solve the equation by completing the square:
$2x^2 + 12x + 3 = 0$

Step 1

If the coefficient of x^2 is 1, then go on to Step 2. Otherwise, divide both sides of the equal sign by the coefficient of x^2.

Since the squared term contains a coefficient other than 1 . . .

Divide every term by 2. . . → $\frac{2x^2}{2} + \frac{12x}{2} + \frac{3}{2} = \frac{0}{2}$ *. . . to get* → $x^2 + 6x + \frac{3}{2} = 0$

Step 2

Get all variable terms on one side of the equation and all constants on the other side.

$$x^2 + 6x + \frac{3}{2} = 0 \rightarrow x^2 + 6x = \frac{-3}{2}$$

Step 3

Complete the square for the resulting binomial by adding the square of half (1/2) the coefficient of the *x* term to both sides of the equation.

a. The coefficient of x *is 6* → $x^2 + 6x = \frac{-3}{2} \rightarrow x^2 + 6x + \left(\frac{6}{2}\right)^2 = \frac{-3}{2} + \left(\frac{6}{2}\right)^2$

b. Divide the 6 by 2 in the parentheses → $x^2 + 6x + (3)^2 = \frac{-3}{2} + (3)^2$

c. Square the resulting 3 and add it to both sides → $x^2 + 6x + 9 = \frac{-3}{2} + 9$

d. Convert 9 to a fraction to add constants on right side of equation → $x^2 + 6x + 9 = \frac{-3}{2} + \frac{18}{2}$

e. Add the fractions → $x^2 + 6x + 9 = \frac{15}{2}$

Step 4

Factor the resulting perfect square trinomial and write it as the square of a binomial.

$$x^2 + 6x + 9 = \frac{15}{2} \rightarrow (x+3)^2 = \frac{15}{2}$$

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Step 5

Use the square root property to solve for x.

a. Take the square root of both sides of the equation . . . → $\sqrt{(x+3)^2} = \sqrt{\frac{15}{2}}$ *. . . to get* → $x+3 = \pm\sqrt{\frac{15}{2}}$

b. Subtract 3 from both sides to isolate the variable → $x = -3 \pm \sqrt{\frac{15}{2}}$

c. This leaves the final answers of → $x = -3+\sqrt{\frac{15}{2}}$ *and* $x = -3-\sqrt{\frac{15}{2}}$

Practice Exercises:

1. $3x^2 - 4x = 4$

2. $y^2 + 8y + 18 = 0$

3. $p^2 + p - 2 = 0$

4. $t^2 - 6t + 3 = 0$

Answers: 1. $\left\{-\frac{2}{3}, 2\right\}$ 2. $\left\{-4+i\sqrt{2}, -4-i\sqrt{2}\right\}$ 3. $\{-2, 1\}$ 4. $\left\{3+\sqrt{6}, 3-\sqrt{6}\right\}$