Solving a System of Three Equations

with THREE VARIABLES each

Example:
$$x + y + z = 2$$
$$x - y + 2z = 2$$
$$-x + 2y - z = 1$$

1. **Choose two of the 3 equations and eliminate one of the variables...**

$$x + \cancel{y} + z = 2$$
$$x - \cancel{y} + 2z = 2$$
$$\boxed{2x + 3z = 4}$$ Box the answer

2. **Now choose two other equations (a different combination), and eliminate the <u>same</u> variable as you did before...**

$$x - y + 2z = 2 \xrightarrow{\text{Multiply by 2}} 2x - \cancel{2y} + 4z = 4$$
$$-x + 2y - z = 1 \qquad\qquad\quad -x + \cancel{2y} - z = 1$$
$$\boxed{x + 3z = 5}$$

Box this answer also

3. **Now take the two <u>boxed</u> answers and use them to eliminate another variable...**

$$2x + 3z = 4 \xrightarrow{\text{Multiply by -1}} 2x + \cancel{3z} = 4$$
$$x + 3z = 5 \qquad\qquad\quad -x - \cancel{3z} = -5$$
$$x = -1$$

4. **Now that you found x, plug it back into one of the boxed equations to find another variable...**

$$(-1) + 3z = 5$$
$$-1 + 3z = 5$$
$$3z = 6$$
$$z = 2$$

5. **Plug the x and z into one of the <u>original</u> equations to find the last variable...**

$$(-1) + y + (2) = 2$$
$$-1 + y + 2 = 2$$
$$y + 1 = 2$$
$$y = 1$$

Solution: (-1, 1, 2)

Solving a System of Three Equations

with a MISSING VARIABLE

Example:
$$x + y + z = 6$$
$$x - y + 2z = 5$$
$$-x \quad\quad -z = -4$$

1. Choose the two equations that are <u>not</u> missing a variable, and eliminate the variable that is missing from the third equation…

$$x + \cancel{y} + z = 6$$
$$\underline{x \cancel{-y} + 2z = 5}$$
$$2x + 3z = 11$$

2. Take the result from step 1 and pair it with the original equation that was missing a variable. Eliminate another variable…

$$2x + 3z = 11$$
$$-x \quad - z = -4$$

Multiply by 3 →

$$2x + \cancel{3z} = 11$$
$$\underline{-3x \cancel{-3z} = -12}$$
$$-x = -1$$

Multiplied by -1 to get a positive x →

$$x = 1$$

3. Now that you found x, plug it into one of the two-variable equations and solve for another variable…

$$-(1) - z = -4$$
$$-1 - z = -4$$

Multiply by -1 to get a positive z →

$$-z = -3$$
$$z = 3$$

4. Plug the x and z into one of the <u>original</u> equations to find the last variable…

$$(1) + y + (3) = 6$$
$$1 + y + 3 = 6$$
$$y + 4 = 6$$
$$y = 2$$

Solution: (1, 2, 3)